Dynamics of Crowds on The Road Between The Two Holy Mosques

Asst.Prof.Dr. Maan A. Rasheed

maan.a@csw.uobaghdad.edu.iq

Asst.Lect .Hiba Ali Kareem

hiba.ali.k@csw.uobaghdad.edu.iq

Department of Mathematics ,College of Science
for Women,University of Baghdad

Abstract

In this paper ,we use one of mathematical crowd models to represent the movement of crowds arriving for the Arbaeen pilgrimage Specifically, we aim to understand the dynamics of crowds on the road between the two holy mosques. Firstly, we introduce the proposed Mathematical crowd motion. In this model, we define a spontaneous speed corresponding to the speed each individual would like to reach in the absence of others. The actual speed is then calculated as a projection of the spontaneous speed onto a set of feasible speeds (i.e., speeds that do not violate the non-interference constraint). Secondly, the numerical computations are conducted using finite difference method and projection techniques. We also perform numerical simulations using an (Open AI, 2025).The obtained results can assist in designing emergency exits, reducing congestion, and improving crowd distribution.

Keywords: Mathematical modeling, Arbaeen pilgrimage, Differential inclusion, Finite difference, The road between the two holy mosques.

Introduction

The human-crowd phenomenon has been a constant concern for experts from various fields. Since the last decade, dynamics of crowds (crowd's behaviors) under normal and emergent situations have been investigated based on modeling simulation technologies (Maury, B., & Venel, J. 2009).Human crowds show complex behaviors, driven by the decisions of individuals, which depend on their environment, goals, obstacles, and interactions with other nearby individuals. The problem of simulating virtual crowds has attracted many researchers recently, see for instance (Yang et al., 2020; Maury & Venel, 2008; Zhou et al., 2010; Narain et al., 2009; Maury & Venel, 2007; Venel, 2009; Løvås, 1994; Degond & Delitala, 2008; Bellomo, Piccoli, & Tosin, 2012), because of its applications in emergency training, architectural design, education and entertainment, urban planning, traffic engineering, policy making,.. etc.

Existing models separate global planning from local collision avoidance and that can simplify modeling.

The local collision avoidance models require that each person take into account the motion of their nearby neighbors. However, for very dense crowds, this step can quickly lead to complicated computational processes

Traditional models (microscopic and macroscopic models) can simulate general crowd dynamics with the advantages of the two types)Maury & Venel, 2007(. To simulate realistic crowds, recent studies of crowds have taken into account the social psychology of crowds. Many other studies on the human crowd and traffic flow can be seen in (Elaiw et al., 2019; Cheng & Li, 2022; Bellomo et al., 2022; Abdulameer & Sarsam, 2014; Sarsam, 2025a, 2025b; Al-Ahbabi & Al-Alwan, 2023; Ghafel & Al-Jawari, 2024).

In (Maury, B., & Venel, J. 2009), the authors proposed a crowd motion model, with a numerical technique. The proposed model depends on two principles: The spontaneous velocity that corresponds to the velocity that each person want to achieve if he is alone. Then, the actual velocity is computed as the projection of the spontaneous velocity onto the set of feasible velocities (velocities that do not allow the non-overlapping). Moreover, they described the new realistic mathematical framework and studied the well-posedness of the model. In addition, they investigated the convergence of the proposed numerical scheme.

The purpose of this paper is to simulate and represent the movement of crowds arriving for the Arbaeen pilgrimage. Specifically, we aim to simulate the dynamics of human crowds on the two-way road between the two holy mosques, which is the place located between the shrine of Imam Hussein and the shrine of Abu al-Fadl al-Abbas in Karbala-Iraq.

During many occasions of the year, particularly during the Arbaeen pilgrimage time, we can see a heavy crowd covering the whole this road. Therefore, it is difficult to assist in designing emergency exits and reducing congestion. The present work can help in understanding the dynamics of crowds on the road between the two holy mosques. This will help us in estimating the required time for people walking in both directions of the road to transition from randomness to structured lanes, while avoiding overlapping and collisions.

For this goal, we consider the simplest model (assuming the same behavior for all people, and they do not use complex ways to escape). Moreover, we apply the Mathematical crowd model proposed in)Maury, B., & Venel, J. 2009(. We solve this model numerically using a numerical scheme, based on finite difference and projection techniques, and we perform the numerical simulations using AI-driven simulations.

The outline of this work is as follows: in the second section, we introduce the proposed crowd motion. In the third section we suggested a numerical technique to solve the model. In section four, we simulate the dynamics of crowds in the a road between the two holy mosques. Based on the obtained results, some observations are pointed out in section five. In the last section, some conclusions and future works are suggested.

Mathematical Crowd Model

In this section, we recall the so called Mathematical crowd model (Maury, B., & Venel, J. 2009).

Consider that we have N people in a domain $\Omega \subset R_2$. Each person (i-th) is identified with a disc of radius r, and center c_i.

The set of configurations is defined as follows:

$$Q = \{q \in R^{2N},\ \ q = (c_1, c_2, \cdots, c_N)\}$$

Also, we assume that denotes the velocity that person (No i) can achieve, if we ignore the existence of others, So we can introduce a spontaneous velocity field as follows:

$$U = (U_1, U_2 \ldots, U_N)$$

With assuming that all people have the same behavior, and they have no complex escaping strategies, a global spontaneous velocity field can be introduced as follows: $U: Q \to R^n$

$$U(q) = (U_0(q_1), U_0(q_2), \ldots, U_0(q_n))$$

Moreover, In this domain, overlapping is not allowed, so,the following set of feasible configurations can be defined as:

$$Q_0 = \{q \in Q, \quad D_{i,j}(q) = |q_j - q_i| > 2r, \quad \forall i \neq j\}$$

As overlapping is not allowed, if two people are in contact with each other, then we can increase distance between them. So, the set of feasible velocities takes the form:

$$V_q = \{v \in R^{2N}, G_{i,j}(q).v \geq 0, when\ , D_{i,j}(q) = 0\}$$

where : $G_{i,j} = (0, \cdots 0, -e_{i,j}, 0, \cdots ,0, e_{i,j}, 0, \cdots ,0) \in R^{2N}$

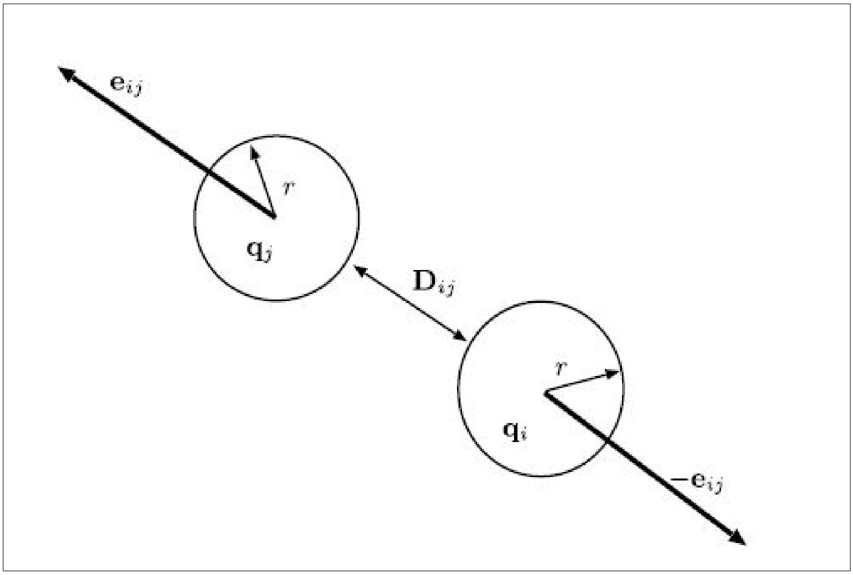


Figure 1 :The distance between two persons

The basic form of the model can be stated as: the actual feasible velocity field is the closest to the spontaneous velocity field, for the Euclidean distance. So the model takes the form:

$$\frac{dq}{dt} = P_{V_q}U(q) \qquad (1)$$

where P_{V_q} refers to Euclidean-projection onto the closed convex cone , and in general the global velocity field can be written as follows:

$$U(q) = (U_1(q), U_2(q), \cdots, U_N(q))$$

where : $U_i(q) = U_0(q_i)$ for i=1,2,…,N.

Thus equation (1) can be written as a system of ODEs as follows:

$$\left.\begin{aligned} \frac{dq_1}{dt} &= P_{V_q}U_0(q_1) \\ \frac{dq_2}{dt} &= P_{V_q}U_0(q_2) \\ &\vdots \\ \frac{dq_N}{dt} &= P_{V_q}U_0(q_N) \end{aligned}\right] \qquad (2)$$

1. Remark

In this work, we limit our scope to the simplest behavioral model. However, we can integrated sophisticated strategies to this approach. For example, the spontaneous velocity of a person may depend upon their abilities such as physical force or personality. In addition, the velocity may be affected by the positions of his neighbors based on social rules or self- optimization strategy.

Although this model is simple, However, it cannot use directly into a standard framework. Therefore, the problem can be reformulated.We introduce N_q , as the outward normal cone to the set of feasible configurations: Q_0 .

Based on the classical orthogonal-decomposition of Hilbert space, and according to Farkas' lemma, equation (1) becomes:

$$\frac{dq}{dt} + N_q \ \ni U(q)\ , \qquad (3)$$

which is a differential inclusion, and it is a generalization of ODEs (1).

For free motion, we get the ODE:

Numerical Technique

This section is devoted to derive a numerical scheme 1, based on finite difference and projection techniques, that is used to solve equation (3),

We divide the time interval (0,T) into m points. That is

$$t_0 = 0,\ t_{n+1} = t_n + h\ , \text{for}\ \ n = 0,1,\dots m-1,\ t_m = T\ ,\ \ h = T/m$$

Let q^n, U^n denote the approximation values of $q(t_n)$, $U(t_n)$ respectively.

$$(q_1^n, q_2^n, \dots, q_N^n)\ , \qquad U^n = (U_0(q_1^n), U_0(q_2^n), \cdots, U_0(q_N^n))$$

By replacing the time derivative at in equation (4) by the Backward finite difference formula, we get:

$$\frac{q^{n+1} - q^n}{h} = U^{n+1}$$

So, we get $\quad q^{n+1} = q^n + hU^{n+1},$

where U^{n+1} minimizes $\frac{1}{2}|v - U(q^n)|^2$ over

$$V_{q^n}^h = \{v, D_{i,j}(q^n) + hG_{i,j}(q^n).v\ \geq 0\}$$

overIt follows that $$U^{n+1} = Proj_{V^h_{q^n}}(U(q^n))$$

The convergence results of this scheme can be found in (Maury, B., & Venel, J. 2009).

Algorithm steps

- Define $t_0 = 0, t_{n+1} = t_n + h$, for $n = 0,1...m-1, t_m + t$, $h = T/m$
- Evaluate the desired velocity: $U^n = \left(U_0(q_1^n), U_0(q_2^n), \cdots, U_0(q_N^n)\right)$
- Put n = 0
- Predict the free motion: $q^{n+1} = q^n + hU^{n+1}$
- Project back onto the constraint set$U^{n+1} = Proj_{V^h_{q^n}}(U(q^n))$, where

  U^{n+1} minimizes $\frac{1}{2}|v - U(q^n)|^2$ over

  $$V^h_{q^n} = \left\{v, D_{i,j}(q^n) + hG_{i,j}(q^n).v \geq 0\right\}$$

- Repeat the steps above for all time levels (n =1,2,...m) until final time T.

Remark 2.

The main computational effort is in computing the projection in the fifth step. That involves solving a convex quadratic program. This can be efficiently done using:

- Gradient projection methods
- Proximal point algorithms
- Newton-type or interior-point metho**ds**

For small N, it can be done directly. For large crowds, more scalable methods (e.g. splitting methods or alternating projections) are needed.

Numerical Simulations of Dynamics of Crowds on the Road Between the Two Holy Mosques

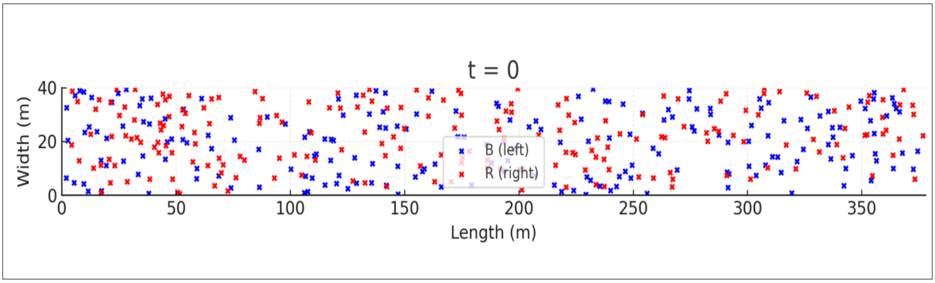
In this section, we apply the simplest model, we proposed in this paper, to simulate the dynamics of crowds on the road between the two holy mosques, which is the place located between the shrine of Imam Hussein and the shrine of Abu al-Fadl al-Abbas, and it is a path 378 m long) WikiShia, 2025(, and 40m width. The total area of the road is . If we assume 5 persons for each 1, then the Road capacity is persons.

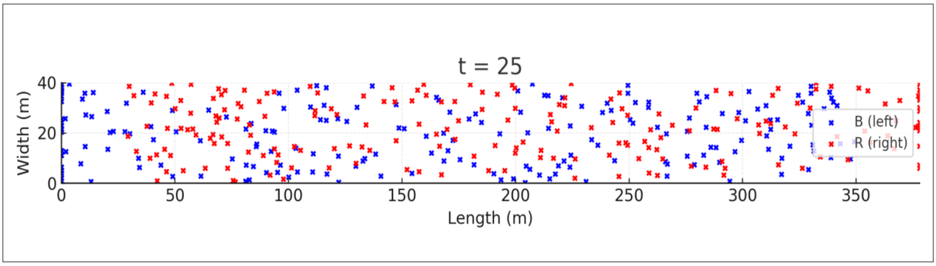
We assume that it is allowed to walk two-way on the road which is bi-dimensional domain. We assume that two populations of people are walking in the road. They are denoted in Blue (B) and Red (R) in the figures. B-individuals want to go to the left, and R-individuals are going to the opposite direction. We assuming the same desired velocity:). The number of individuals in each population is 200 (N=400).

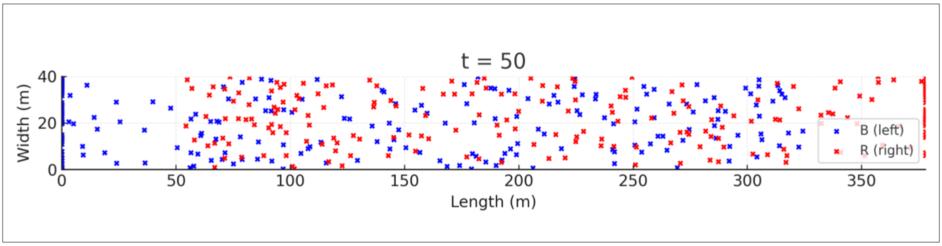
In order to present the numerical simulations of this problem, we apply the numerical algorithm, presented in the last section, and due to the complexity of the problem, we use an (Open AI, 2025) in the computations and simulations processes.

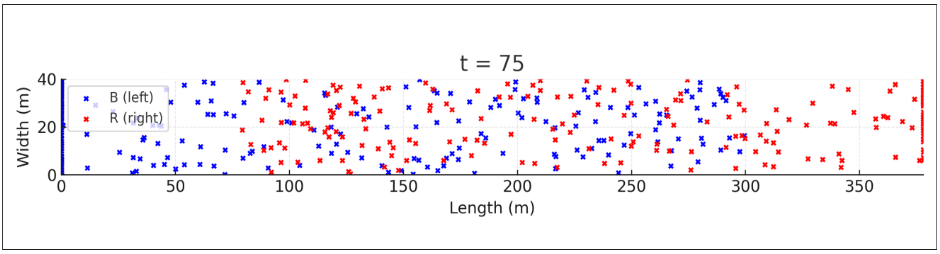
Figure 2. shows the two populations (R and B), at times (in seconds) 0 (random distribution), 25, 75, 50, 75, 100, 125, 150, 175, and 200 (T=200), based on the proposed numerical algorithm.

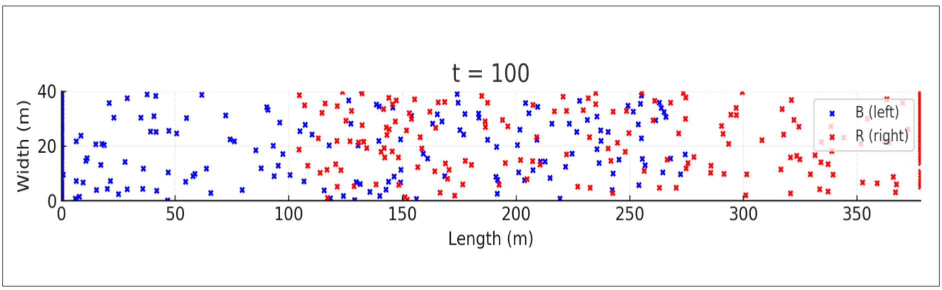
Road between the two holy mosques

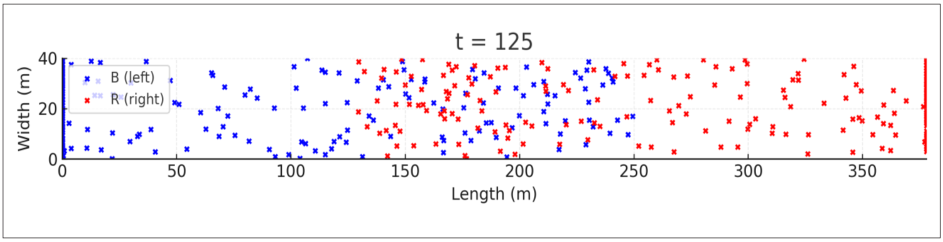

t = 0
Width (m)
Length (m)
0
20
40
0
50
100
150
200
250
300
350
B (left)
R (right)

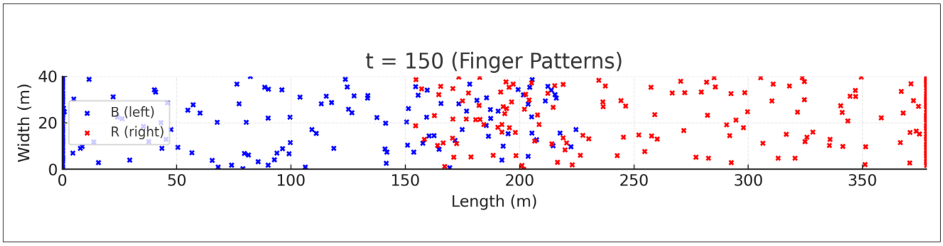

t = 25
Width (m)
Length (m)
0
20
40
0
50
100
150
200
250
300
350
B (left)
R (right)

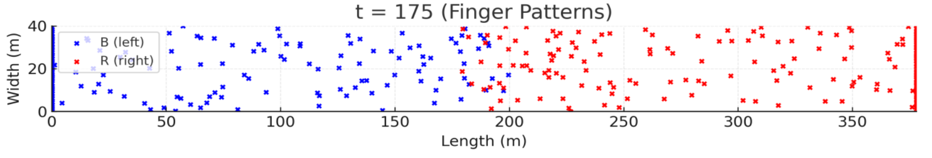

t = 50
Width (m)
Length (m)
0
20
40
0
50
100
150
200
250
300
350
B (left)
R (right)


t = 75
Width (m)
Length (m)
0
20
40
0
50
100
150
200
250
300
350
B (left)
R (right)


t = 100
Width (m)
Length (m)
0
20
40
0
50
100
150
200
250
300
350
B (left)
R (right)


t = 125
B (left)
R (right)
Width (m)
Length (m)


t = 150 (Finger Patterns)
B (left)
R (right)
Width (m)
Length (m)


t = 175 (Finger Patterns)
B (left)
R (right)
Width (m)
Length (m)

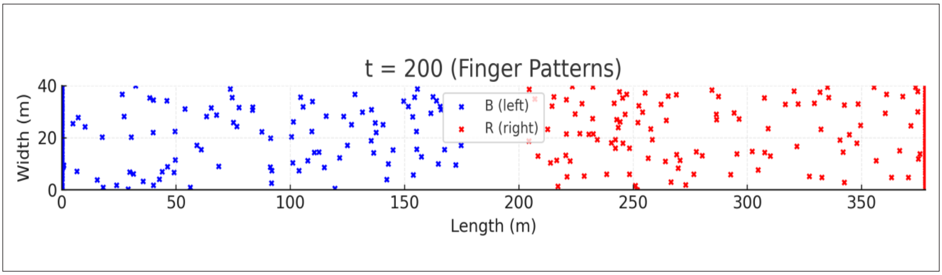

t = 200 (Finger Patterns)
B (left)
R (right)
Width (m)
Length (m)

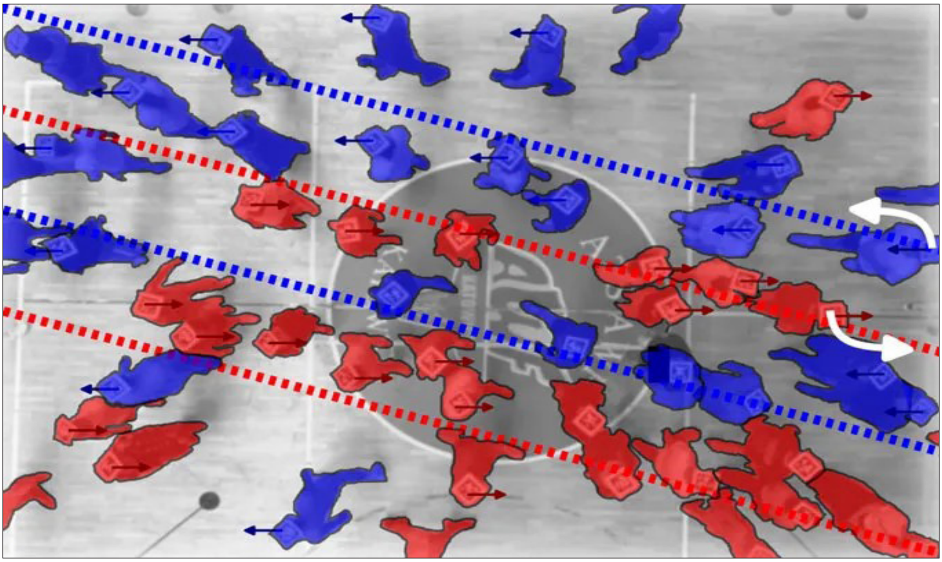
Figure 2.

Numerical Simulation showing the movement of the two populations over time

Observations

Based on Figures 2, we can point out the observations:

1. Individuals repel each other when they get too close, so clustering and overlapping are avoided.
2. To avoid collisions, people spread more across the road width (y-axis), especially in the central region where the two flows interact.
3. Over time, the system transitions from randomness to structured lanes, illustrating the emergence of order from simple rules.
4. Despite repulsion and spatial constraints, desired directionality (left for B, right for R) is maintained throughout the simulation. This illustrates the model's stability in preserving macroscopic flow goals while resolving microscopic constraints.
5. At later times (t=150,175 and 200), there are clearer alternating spatial streams forming where Blue and Red populations interact. These narrow, elongated paths are characteristic of the so-called finger patterns)Helbing & Molnar, 1995(. The populations start to self-organize into structured, side-by-side "fingers", reducing direct frontal collisions.



Finger Patterns

Conclusions and Future Studies

Based on the numerical simulation results obtained in the last section, we can understand the dynamics of crowds during the Arbaeen Pilgrimage time, specifically on the road between the two holy mosques. The numerical simulation successfully captures the emergence of finger patterns, ensures feasibility under rigid disc constraints, and exhibits collective behavior consistent with bidirectional pedestrian flows.

The novelty of this work is that it uses AI to simulate the governed problem based on the proposed mathematical crowd model and the numerical technique. The obtained results can assist in designing emergency exits, reducing congestion, and improving crowd distribution, and estimating the required time for people walking in both directions of the road to transition from randomness to structured lanes, while avoiding overlapping and collisions.

As possible future works, we can integrated sophisticated strategies to this approach. For example, the spontaneous velocity of a person may depend upon their abilities such as physical force or personality. In addition, the velocity may be affected by the positions of his neighbors based on social rules or self-optimization strategy.

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