Employing The Traveling Salesman Problem in Solving the Holy Shrines Visitor Problem in Iraq to Determine the Shortest Path

Asst.Lect.Sajjad Majeed Jasim

Dept. of Mathematics, College of Science, Mustansiriyah University.

sajadm49@gmail.com

Asst.Prof.Manal Ghassan Ahmed

Dept. of Mathematics, College of Science, Mustansiriyah University.

manalghassan@uomustansiriyah.edu.iq

Prof. Faez Hassan Ali

Dept. of Mathematics, College of Science, Mustansiriyah University.

faezhassan@uomustansiriyah.edu.iq

Abstract

Visiting the holy shrines is of great importance to Muslims in Islamic countries in general and to Muslims in Iraq in particular. Visitors usually try to visit as many holy shrines as possible in a single visit at the lowest possible cost. In this research, visiting the Iraqi Holy Shrines Problem (IHSP) was considered, which is in fact, a practical application of the symmetrical traveling salesman problem (TSP). The summary of this problem is that a visitor wants to visit all (or most) of the (14) holy shrines in Iraq. This visitor lives in a city that includes one of these shrines and wants to find the appropriate route to visit all of them at the lowest cost (cost here means distance and/or time and/or financial cost) and then return to the same city from which he started without repeating the route between any two shrines. One of the most important results of this research is that the optimal (shortest) distance path was found to help the visitor achieve the minimum financial cost of fuel consumption using more than one accurate or approximate method to solve this problem.

Keywords: Holy shrines in Iraq, Combinatorial Optimization Problem Traveling Salesman Problem, Graph theory, Branch and Bound Method, Genetic Algorithm.

Introduction

Iraq holds a unique and prominent position in the Islamic world due to its rich religious, historical, and cultural heritage. It is home to some of the most revered and visited holy sites in Islam. The country's landscape is dotted with shrines, mosques, and historical sites that date back to the earliest centuries of Islamic history. These sacred places are not merely architectural landmarks but are deeply embedded in the collective memory and religious practices of millions of Muslims around the world. The Travelling Salesman Problem (TSP) is a classical problem in combinatorial optimization. It involves finding a Hamiltonian cycle of minimum total weight in a weighted graph — that is, a cycle that visits each vertex exactly once and returns to the starting vertex. The TSP is known to be NP-hard, meaning that no polynomial-time algorithm is currently known to solve all instances of the problem efficiently (Xiang, et al., 2015). The problem of visiting a number of holy shrines within a specific geographical area can be treated as a TSP, which has different geographical, economic, engineering, electronic applications, etc.

Over the past decades, there have been many previous studies that addressed the TSP. We will attempt to review the most important and recent of these studies. In their study (Jasim & Ali, 2018) investigated several heuristic techniques for solving the TSP, including the Minimizing Distance Method (MDM), Branch and Bound Method (BABM), Greedy Method (GRM) and the Tree-Type Heuristic Method (TTHM). They identified the limitations of the classical MDM and proposed an enhanced form, the Improved Minimizing Distance Method (IMDM), which showed better performance compared to the original and other heuristics. The GRM, on the other hand, achieved the best computational efficiency in terms of execution time for instances where the number of cities ranged from

5 to 500. (Jasim & Ali, 2019a) employed the TSP as a model to compute the minimum possible travel cost (distance or time) across a network of Iraqi cities. The study focused on two main solution methods: BAB Technique (BABT) and Dynamic Programming (DP). To obtain the best BABT, more than one upper (UB) lower (LB) bound and are derived. The outcomes of each approach are exactly the same, with a shorter delay for the number of cities (). These results demonstrate the effectiveness of BABT in comparison with a few good heuristic methods. (Jasim & Ali, 2019b) investigates some exact (like BABT) and local search methods (LSM's) like classical Simulated Annealing (SA) and Genetic Algorithm (GA) to solve the TSP. The GA is improved by two kinds of improvements for GA; the 1st one Hybrid GA (HGA) and the 2nd improved GA (IGA). In 2021, (Garn, 2021) presented two heuristic approaches to solve the balanced dynamic MTSP (B−MTSP) The techniques serve as tactical tools for real-time or dynamic (online) routing. Different types and ranges of dynamics are proposed. For such dynamics, the BD assignment vehicle heuristic (BD-AVH) and the BD closest vehicle heuristic (BD-CVH) are used. In the Euclidean plane, the proposed techniques are tested for many different examples. As strategic instruments for dynamic routing, continuous models for the BD-MTSPs are derived. Without applying an algorithm, the proposed models clarify the route lengths dependent on customers, dynamic scopes and vehicles. (Ahmed & Ali, 2022) found the most effective solutions for Multi-Criteria TSP (MCTSP) by using two local search methods (LSMs): the Bees Algorithm (BA) and Particle Swarm Optimization (PSO). The PSO and BA results are compared to the results of the BAB and complete enumeration methods (CEM), in addition some heuristic techniques. Results showed that PSO and BA methods were effective for a large number of nodes ($n \leq 700$). Two criteria; distance

and time are proposed by (Ahmed & Ali, 2023) for Multi-Criteria TSP (MCTSP). They suggest new methods to solving MCTSP, both heuristic and exact. The MCTSP's mathematical formulation was covered in the theoretical section. The Branch and Bound (BAB) technique, which solves MCTSP for ($n \leq 40$) in a reasonable of time, was suggested in the practical section along with new upper and lower bounds. In contrast, they suggest four heuristic methods that achieved good results when compared to exact methods. The results of this study demonstrate the effectiveness these techniques are at solving MCTSP. The Multiobjective TSP (MOTSP) with weights is investigated in the paper by (Ahmed , Ali, Khalaf , & sabri Al-Safi, 2023). They suggested two heuristic approaches, MDA and MDTM, to identify the best solution and employed the BAB method to do so. To demonstrate the effectiveness of these approaches, they lastly contrasted the heuristic approaches with BAB. Results that show the effectiveness of these techniques work.

In the rest of this paper, Section 2 presents some fundamental TSP concepts. In section 3 we introduce the most important methods for solving TSP for last five years. While in section 4, the vehicle consumption of gasoline will be discussed. Important introduction about the Holy Places in Iraq are introduced in section 5. While the problems which are faced by the visitors to the Holy Shrines are discussed in section 6. In section 7, the Iraqi holy places visitor's problem is discussed theoretically. In section 8, we applying the solving methods for IHSP. Lastly, in section 9, we will introduce some conclusion and future work.

Travelling Salesman Problem Concept

TSP is made up of a set of nodes (say), with a path connecting any two of them. The cost of the pair of nodes is known, and each of these paths has a specified cost. Traveling salesman begins at a specific node and proceeds to all other nodes without returning to the same node, eventually returning to the origin node. The primary goal of TSP is to find the complete path that minimizes total cost during movement (Hosseinabadi, yazdanpanah, & Rostami, 2012).

The main example of TSP is the Global Positioning System (GPS) that is often used by the drivers in order to navigate and show them the shortest route to an unfamiliar place. Consider the amount of devices in a production line. The primary function of these machines is to drill various holes in a piece of material. This material could be a board of circuits, a car frame, or a piece of wood for a bookcase. The drill may travel to any point within a designated region through motors that follow tracks. Finding a solution to the TSP could be useful to find the optimal or best order in which all the holes should be drilled (Zambito, 2006).

Mechanical Engineering or electronic connection position is another application for which a TSP solution may be useful. Take into consideration the electrical wiring of a large building, the plumbing design of a building, or even the wiring of a particular circuit board. In a few of these examples, the connections must be arranged so that every element is connected in a cycle (Taha, 2011).

1.Mathematical Formulation of TSP:

TSP model depends on a number of cities (say n) and we may define the distance matrix ||dij||,i,j=1,…,n (d_ii=0 or ∞) to represent the cost between cities i and j. The matrix X that we defined may be described as the following (Taha, 2011):

Then the Matrix is:

$$X = \begin{bmatrix} 0 & x_{12} & \cdots & x_{1n} \\ x_{21} & 0 & \cdots & x_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ x_{n1} & x_{n2} & \cdots & 0 \end{bmatrix}$$

The TSP is considered as a linear programing problem so the mathematical formulation is as follows:

$$\text{Minimize } z = \sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}x_{ij}$$

Subject to:

$$\sum_{j=1}^{n} x_{ij} = 1, i = 1,\ldots,n$$

$$\sum_{i=1}^{n} x_{ij} = 1, j = 1,\ldots,n$$

$$x_{ij} = 0, 1$$

Example (1):

For TSP, suppose we have the following distance table for 5-cities:

	A	B	C	D	E
A	0	5	7	4	9
B	5	0	10	12	3
C	7	10	0	6	8
D	4	12	6	0	10
E	9	3	8	10	0

Let's start from city (A), and suppose we have the rout A-C-D-E-B-A which obtained from any solving method, so the matrix X will be:

$$X=\begin{bmatrix}0&0&1&0&0\\1&0&0&0&0\\0&0&0&1&0\\0&0&0&0&1\\0&1&0&0&0\end{bmatrix}$$

Notice that each row and column has only one (1), obtain the cost value Z:

$Z=d_{13}+d_{34}+d_{45}+d_{52}+d_{21}=7+6+10+3+5=31$ units.

2.Successive Rules and Reduction for TSP (Jasim, 2019):

We can define the Successive Rules (SR) by the rules which are enforcing the obtained sequence or path to be arranged in some specific order. These SR may be direct (e.g B→C) or indirect (e.g B→...→C). If the TSP has some mandatory , it might be applied directly. In solving the COP, the is extremely important, especially TSP. The will help in the reduction of the number of cities, therefore the problem's size is reduced, which means that less computing time is needed to solve it. After reduction, the size of the matrix is (n-m)×(n-m) if the TSP matrix is (n×n) in size and there are (m) SRs.

Remark (1): The two types of TSP which are symmetric and asymmetric matrices when reducing its matrix will be transformed into an asymmetric matrix.

Solving Methods for TSP

1.Exact Methods for Solving TSP:

In this paper we will focus on branch and bound techniques, dynamic programming, and complete enumeration, among other methods to exact solution.

A. Complete Enumeration Method:

(CEM) the most general method of problem solving, also referred to as general and test, involves methodically listing every potential solution and evaluating each one to determine which approach best solves this problem. One of the most well-known techniques for solving TSP is this one (Dahiya & Sangwan, 2018).

B. Dynamic Programming (DP):

is a technique for problem-solving that divides the solution into a number of stages or steps so that the problem-solving process can be seen as a sequence of linked decisions. Richard Bellman is the creator and driving force behind DP's popularity. DP uses the optimality principle to make a series of optimal decisions. The optimality principle states that if the complete solution is optimal, the solution to the kth stage must also be optimal. The concept of optimality ensures that at some point in the decision-making process, the correct decision is made for subsequent stages. The basic principle of DP is removing just a portion of a problem at each step, then overcome minor issues and use the settlement results to correct the problem in the following step (Nugroho, 2010).

C. The Branch and Bound Technique (BABT)

It is usually used in COP, in particular TSP, where it builds a state space tree to determine the best value for the objective function among all viable solutions. Dantzig initially studied BAB and provided a more detailed description of it in TSP applications. The BABT provides every feasible option by solving the problem and attempting the practical solution with a value in the upper bound to find the most suitable one (Ranjana, 2018) .

2.Approximation Approaches for Solving TSP:

In general, the term "heuristics" refers to algorithms that search to identify solutions from all of the possible choices. Though heuristic algorithms often discover a solution that is quick and simple, they do not always ensure that the best or optimum solution will be found. These heuristics might at times be accurate, that is, they may identify an optimal solution; but the algorithm remains a heuristic algorithm until the optimal solution has been found (Lawler, Lenstra, Rinnooy Kan, & Shmoys, 1985).

A. The BAB Method (BABM)

reflects all state space search methods in which all children of an E-node are generated as any now nodes known as live nodes when it becomes an E-node. E-node is a node that can be expanded. The live node is a node that has been generated but has not yet had its children expanded. Dead nodes are nodes that cannot be expanded, but they can be useful for the backtracking concept. If there are no more children to expand, we must return to its parent and expand its children, continuing until we reach the solution or accomplish tree path (Rastogi, Shrivastava, Payal, & Singh, 2013). It's important to note that this method is completely different from BABT.

B. The Minimizing Distance Method (MDM)

It is one of the most effective techniques discussed above in terms of outcomes; to reduce the matrix, we must subtract the smallest element from each row and each column until there is zero in each row and column. The penalties are then determined for each zero, and the smallest number in each zero's row and column is collected. We link the generated routes and compute the cost after deleting the row and column that contain zero and canceling the item with the reverse location of zero of the matrices. This procedure is repeated until the dimension of the reduced matrix is 2 (Swarup, Gupta, & Mohan, 1994).

3.Local Search Methods for Solving TSP:

A. Local Search Methods (LSMs)

will ultimately have an important effect on recent studies in computer science because they aid academics solve issues across a number of areas for solutions that in their full generalization do not exist in a timely manner, even with the world's fastest computers, as well as being universal search heuristics and simple to implement (Tsai, Tseng, Chiang, & Yang, 2014). A metaheuristic may improve a complicated issue by searching through a large number of possible solutions with only a few predictions about the problem at hand and no guarantee of finding the best solution. Some metaheuristics take either a single solution-based or a population-based approach (Srour, Othman, & Hamdan, 2014).

B. Genetic Algorithms (GA)

is Holland's derivative free stochastic technique, which is based on biological evolutionary processes. The following generation should be healthier and fitter than the previous one since in nature, individuals who are best suited will likely to survive and mate. In a book that Golberg regularly cites, a great deal of effort and applications have been made about GAs. The population of chromosomes that GAs work with can be defined by a set of fundamental parameters (Hussain, et al., 2017).

We suggested using new crossover called Mixing Crossover Mutation Algorithm (MCMA) for GA to obtain Improved GA.

C. Mixing Crossover Mutation Algorithm (MCMA)

(ch_1, ch_2, ch_3, ch_4)=MCM (ch)

ch_1=Simple inversion crossover (ch);

ch_2=Swap (ch);

ch_3=Displacement (ch);

ch_4=ch;

end;

The Vehicle Consumption of Gasoline (Power, 2024)

Vehicle Consumption of Gasoline refers to the amount of gasoline a vehicle uses to travel a certain distance or operate over a certain time. It is a key factor in understanding fuel efficiency, environmental impact, and operating costs of a vehicle.

We usually have two types of gasoline: regular gasoline (also known as 90 or 91 octane gasoline, depending on the country) and enhanced or premium gasoline (95 octane gasoline). Fuel consumption varies depending on several factors, including:

1. Vehicle type (small, medium, SUV, pickup, sports car, etc.).
2. Year of manufacture.
3. Engine type (gasoline, diesel, hybrid, partial electric, etc.).
4. Driving style (slow or aggressive, city or highway).
5. Vehicle condition (filter cleanliness, tire pressure, etc.).

Table (1) shows fuel consumption rates according to the type of vehicle and the type of gasoline

Table (1): Fuel consumption rates according to the type of vehicle and the type of gasoline.

Vehicle	Type of Vehicle	Consumption rates of gasoline L/100	
		Regular (Re)	Premium (Pr)
	Hybrid Car (HC)	3-5	4-5
	Small Car (SC)	5-7	5-7
	Mid-Range Sedan (MC)	6-9	7-9
	SUV or 4WD (BC)	8-14	9-14
	Sports Car (SC)	10-18	12-18

Here we will take the worst case s.t. we will take the upper level (UL) of range of Consumption rates of gasoline (CRG). As seen in table (1), that UL in each Km is UL/100, for example for HC with regular or premium the consumption rate in each Km is CR=0.05. As we know that the price (P) for each liter of Regular (PR) is 450 ID and for Premium (PP) is 850 ID, so the cost (C) of one Km for any vehicle in ID is:

C=CR*P ....(3)

Where P=PR or PP.

Where or.

Table (2) shows the travel cost according to the type of vehicle of in .

Table (2): The travel cost according to the type of vehicle of in .

Vehicle	Type of Vehicle	of in	
		Regular	Premium
	HC	22.5	42.5
	SC	31.5	59.5
	MC	40.5	76.5
	BC	63.0	119.0
	SC	81.0	153.0

Holy Places in Iraq (BBC,2024)

Among the most important of these are the shrines of the Prophet Muhammad’s family (Ahl al-Bayt), such as those of Imam Ali in Najaf, Imam Hussain and Al-Abbas in Karbala, and the Imams in Samarra and Kadhimiya. Each of these locations holds profound spiritual significance and is associated with major events in Islamic history, especially the martyrdom of key religious figures, which are commemorated annually through large-scale pilgrimages and rituals.

1.Significance of the Holy Sites:

- **Religious :**These shrines serve as spiritual centers that reinforce religious devotion and commemorate major Islamic events.
- **Social :**They bring together people from various ethnic and national backgrounds ,fostering social and cultural unity.
- **Economic :**Religious tourism boosts local economies through hotels, transportation ,markets ,and public services.
- **Cultural and Political :**These sites are integral to Iraq‘s historical and religious identity ,symbolizing centuries of tradition and spiritual heritage.

2.Number of Visitors Annually:

- Official and independent estimates suggest that Iraq's holy shrines receive over 30 million visitors annually.
- The largest numbers are seen during Arbaeen, with around 20 million pilgrims visiting Karbala, including 3 to 5 million international visitors from countries.

The shrines and holy sites in Iraq represent some of the most significant religious landmarks in the Islamic world. They have a deep impact on Iraq's spiritual, cultural, and social life. Furthermore, these sacred places serve as a point of convergence for millions of visitors each year, highlighting Iraq's central role in global religious tourism.

The Problems Faced by Visitors to the Holy Shrines

As we mentioned earlier, Iraq contains many holy shrines that include the tombs or mausoleums of prophets, Imams of the Household of the Prophet (peace be upon him and his family), companions of the Prophet Muhammad (peace be upon him and his family), or companions of the Imams of the Household of the Prophet (peace be upon them). Visitors to these holy shrines, especially those from outside Iraq, may wish to visit all or most of these holy sites, because such a visit may not be possible twice, in the shortest possible time, over the shortest distance, and at the lowest possible cost.

The most important problems that a visitor who wants to visit all the holy sites may face can be summarized as follows:

1. Some of these holy sites are located within a single Iraqi governorate. Fortunately for the visitor, some holy sites contain more than one shrine or tomb at the same time. But the main problem is that some of the other holy sites may be located in other cities or governorates that may not be close to the rest of the holy sites.

2. This burdens the visitor, as he may be elderly, suffer from certain diseases that prevent him from traveling for a long period.
3. The visitor may have limited financial and material capabilities.
4. The visitor may does not have enough time to visit all the holy sites due to work commitments or other reasons.
5. Visitors may arrive in their own vehicle and wish to visit all, or at least some, of these holy sites, rather than tour company, that mean they must adhere to all the instructions imposed by the tour company.
6. Since the visitor will be arriving in their own vehicle, they will need to refuel frequently to ensure a complete visit. This, of course, will incur additional costs, this means if they have the shortest path, it means the lowest cost.

Iraqi Holy Shrine Problem

The Iraqi Holy Shrine Problem (IHSP) is a symmetric TSP. The most important Holy Shrine in Iraq consists of 14 shrines, the travelling cost between each two shrines is known. A visitor lives in one city of the shrine's cities (say Najaf), he wants to visit all holy shrine starting from his city, then he goes to all other holy shrines without repeating the path between any two shrines, lastly hew will return to his city. We wish to help him (her) to find the minimum total cost of these shrines starting from his city. The symbol of each shrine is as in table (3).

Table (3): The symbol of each holy shrine in Iraq.

	Holy places	Governorate	City	Symbol
1	Imam Ali and Kufa	Najaf	Najaf	AK
2	Imam Hussein	Karbala	Karbala	HS
3	Al-Kadhimayn	Baghdad	Kadhimiya	KA
4	Salman Al-Muhammadi		Madain	SM
5	Al-Askariyyan	Salah Al-Din	Samarra	AS
6	Sayed Muhammad Sab' Al-Dujail		Balad	SD
7	Imam Al-Qasim son of Al-Kadhim	Babylon	Al Qasim	QK
8	Sharifa bint Al-Hasan		Hillah (Griq)	SH
9	Sons of Muslim ibn Aqil and Al-Qasim ibn Al-Hasan		Musayyib	SQ
10	Zayd ibn Ali Ash-Shaheed		Hillah (Al-Kifl)	ZS
11	Saeed ibn Jubair	Wasit	Al-Hayy	SJ
12	Prophet Uzair (Azir)	Maysan	Al Uzair	UZ
13	Prophet Shu'ayb	Al-Qadisiyah	Diwaniya	SB
14	Zain Al-Abidin	Kirku	Daquq	ZA

In this section, we will employ the TSP as a tool to calculate the least total cost for Iraqi shrines. First, we show the distance in kilometers (km) as shown in table (4) for the shrines (Google, 2025).

Table (4): The distance (Km) between the Holy places.

Governorate		Sym	AK	HS	KA	SM	AS	SD	QK	SH	SQ	ZS	SJ	UZ	SB	ZA
	Sym	Num	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Naj	AK	1	-	80	190	173	306	270	75	63	110	45	220	378	165	407
Kar	HS	2	80	-	120	113	226	189	83	50	40	71	252	430	142	336
Baghdad	KA	3	190	120	-	53	116	80	164	135	91	160	235	442	187	225
	SM	4	173	113	53	-	163	127	147	118	75	146	213	420	171	254
Salah-AlDin	AS	5	306	226	116	163	-	54	277	250	195	280	347	554	304	164
	SD	6	270	189	80	127	54	-	244	224	159	243	311	518	267	197
Babylon	QK	7	75	83	164	147	277	244	-	44	92	67	231	406	57	381
	SH	8	63	50	135	118	250	224	44	-	56	45	221	399	111	353
	SQ	9	110	40	91	75	195	159	92	56	-	91	248	426	138	314
	ZS	10	45	71	160	146	280	243	67	45	91	-	253	401	140	385
Was	SJ	11	220	252	235	213	347	311	231	221	248	253	-	219	256	437
May	UZ	12	378	430	442	420	554	518	406	399	426	401	219	-	463	688
Diw	SB	13	165	142	187	171	304	267	57	111	138	140	256	463	-	402
Kir	ZA	14	407	336	225	254	164	197	381	353	314	385	437	688	402	-

Applying Solving Methods for IHSP

In this section we exploit the TSP to evaluate the minimum total cost (distance or cost) for IHSP. So three types of solving methods are investigated to solve this problem:

1. Heuristic Methods: IMDM and BABM.
2. LSM: GA and IGA.
3. Exact Methods: DP and BABT:

Table (5) shows the results of applying the above Heuristics, LSM and Exact Methods of solving HISP for.

Table (5): the results of applying Heuristic, LSM and exact Methods of solving HISP for.

Type of Method	Method	D/Km	T/s	Path
Heuristic	BABM	1893	0.5	1 ,10, 8, 2, 9, 4, 3, 6, 5, 14, 13, 7, 11, 12, 1
	IMDM	1866	0.6	1, 10, 8, 7, 13, 9, 2, 14, 5, 6, 3, 4, 11, 12, 1
LSM	GA	1864	1.5	1,10, 8, 7, 13, 9, 2, 6, 5, 14, 3, 4, 11, 12, 1
	IGA	1823*	0.3	1, 12, 11, 4, 14, 5, 6, 3, 9, 2, 8, 13, 7, 10, 1
Exact	DP	1823*	600	1, 12, 11, 4, 14, 5, 6, 3, 9, 2, 8, 13, 7, 10, 1
	BABT	1823*	5	1, 12, 11, 4, 14, 5, 6, 3, 9, 2, 8, 13, 7, 10, 1

The symmetric path for the above optimal costs, (referred to with *), is as follows:

1	12	11	4	14	5	6	3	9	2	8	13	7	10	1
AK	UZ	SJ	SM	ZA	AS	SD	KA	SQ	HS	SH	SB	QK	ZS	AK
Naj	May	Was	Baghdad	Kir	SalahAldin		Baghdad	Babylon	Kar	Babylon	Qad	Babylon		Naj
Naj	Uzair	Hay	Madain	Daq	Sam-Balad		Kad	Musayab	Kar	Hillah	Diw	Qas-Kifl		Naj

Remark(2): From the above optimal path we notice that Baghdad is separated in travel, if we modified this path to join the two shrines in Baghdad in one path we obtain the following path:

1	12	11	4	3	14	5	6	9	2	8	13	7	10	1
AK	UZ	SJ	SM	KA	ZA	AS	SD	SQ	HS	SH	SB	QK	ZS	AK
Naj	May	Was	Baghdad		Kir	SalahAldin		Babylon	Kar	Babylon	Qad	Babylon		Naj
Naj	Uzair	Hay	Mad-Kad		Daq	Sam-Balad		Musayab	Kar	Hillah	Diw	Qas-Kifl		Naj

This path gives total distance and its close to the optimal path.

As we see from table (5), the minimum results for optimal distance (OD) is 1823 , so the optimal travel total cost (OTTC) according to the type of vehicle and type of gasoline and depending on relation (2) is shown in table (6).

Table (6): The O for according to the type of vehicle and gasoline in .

	Type of vehicle	in	
		Regular	Premium
	Hybrid Car (HC)	41,018	77,477
	Small Car (SC)	57,424	108,468
	Mid-Range Sedan (MC)	73,831	139,459
	SUV or 4WD (BC)	114,849	216,937
	Sports Car (SC)	147,663	278,919

Conclusion and Future Work

1. In this paper we will applying the principle of TSP to solve IHSP by applying (3) types of solving methods with (6) methods.
2. Table (5) shows that, for heuristics: the IMDM is better than BABM, for LSM: the IGA (with optimal value) is better than GA, while for exact: BABT and DP give the same optimal value but BABT is better in CPU-time since it solves the IHPS in 5 sec.
3. In table (5), we help the visitor to obtain the minimum distance () to visit the most important (14) holy places in Iraq.
4. The minimum distance means the minimum cost to travel staring from Najaf city to other (9) Iraqi governorates to visit the holy places according to type of vehicle and gasoline (see table (6)).
5. We think that if we change the staring city that mean change in the minimum total distance and cost.
6. As future work, we can add more holy places to the list of total holy places in Iraq and calculate the suitable path to obtain the minimum distance and cost.
7. This problem considered a single objective function, we suggesting to develop the problem to multi-criteria objective function solve the problem with two or three objectives simultaneously.

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